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Mathematical Modeling for Modal Computations Concerning a Morphable Wing

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I. Introduction

TYPICAL military and civilian aircraft are subject to widely varying physical configurations and freestream conditions over a given mission-specific flight profile. As a result, an infinite number of discrete conditional points can be identified for possible optimal solutions when laying out the final design of the aircraft. In the case of an aircraft with conventional fixed-wing geometry, a single design point must be selected and used for all subsequent optimization studies—thus necessitating suboptimal performance at off-design points along the mission flight regime.

The use of adaptive-wing technology has been identified as a possible method of reducing and/or eliminating the adverse effects of off-design aircraft performance. By continually varying the geometry of the wing, the chordwise and spanwise camber distributions can be modified such that an optimal solution for the configuration of the wing is admitted, given the current freestream conditions and aircraft weight. Investigations performed by Szodruch[1] and Siclari et al.[2] for the wings of large civil transport aircraft indicate that significant drag reductions can be realized through the use of variable leading- and trailing-edge camber systems. For high-speed flight, similar advantages of variable camber high lift devices were noted by Smith and Nelson[3] in their analysis of the ATFI/F-111. Such results are promising; however, to date the idea of an aircraft possessing continuously variable wing geometry has not come to fruition.

The task of developing and implementing an adaptable wing structure for the "in-service" environment is far from trivial. The familiar problems of structural design, aerodynamics, stability, and control must be redefined in terms of the continuous variability of the wing geometry during flight. Additionally, the strain-energy magnitudes encountered during the geometry modification of a conventional wing in flight have placed increased demand on the development of more robust actuator and mechanistic technologies.[4] For a thorough discussion of novel actuating techniques, the reader is referred to Monner[5] and Campanile and Sachau.[6]

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This Note serves to offer an analytical method of describing the evolution of the natural frequencies and mode-shapes characteristic of an adaptable wing during an arbitrary actuation cycle. Although inherently simplified, the resulting method is seen to be well suited for direct application in the theoretical analysis of the aeroelastic stability of such structures.

II. Motivation/Objectives

Although a thorough treatment of aeroelastic stability, with regard to a wing structure, would necessitate consideration of both static and dynamic phenomena, the present discussion will focus singularly on aerodynamic self-excited oscillations—specifically, bending-torsion flutter. Such a route is justified by considering the magnitudes of wing critical speeds as presented by Barmby et al.,[7] Smilg and Wasserman,[8] and Bisplinghoff and Ashley.[9]

Assuming a fixed aileron with no deflection, Lagrange's equation[10] for the idealized pitch-plunge aerofoil section of Theodorsen[11] and Theodorsen and Garrick[12] can be written as

$$m\frac{\mathrm{d}^2h}{\mathrm{d}t^2} + mr_{\mathrm{c.g.}}\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} + k_b h = -L \tag{1}$$

and

$$mr_{\mathrm{c.g.}}\frac{\mathrm{d}^2h}{\mathrm{d}t^2} + I_\alpha\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} + k_\theta\alpha = M \tag{2}$$

where the variables in Eqs. (1) and (2) are identified as follows m is the mass of the aerofoil section, $mr_{\mathrm{c.g.}}$ is the static moment of the aerofoil section about the axis of rotation, I_α is the aerofoil section mass moment of inertia about the axis of rotation, k_b is the effective sectional bending stiffness, k_θ is the effective sectional torsional stiffness, L is the aerodynamic lift, M is the aerodynamic moment, and h and α are the generalized coordinates of pitch and plunge, respectively.

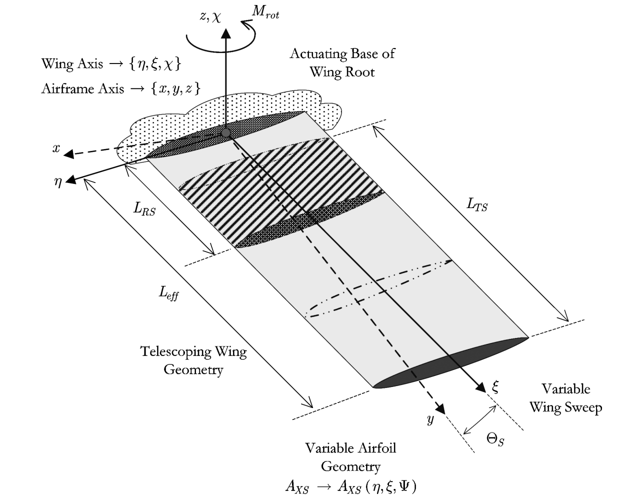
The motivation for the present work can be seen by considering the uncoupled, "free" response of the given system by setting $L = M = mr_{\mathrm{c.g.}} = 0$, where the tertiary condition is satisfied physically by letting the axis of rotation coincide with the center of gravity of the aerofoil section. With these substitutions, Eqs. (1) and (2) are seen to reduce to the form of a pair of uncoupled harmonic oscillators, with bending and torsional natural frequencies of $\omega_b = \sqrt{(k_b/m)}$ and $\omega_\theta = \sqrt{(k_\theta/I_\alpha)}$, respectively. Substituting this result into our original system of equations yields

$$m\frac{\mathrm{d}^2h}{\mathrm{d}t^2} + mr_{\mathrm{c.g.}}\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} + m\omega_b^2 h = -L \tag{3}$$

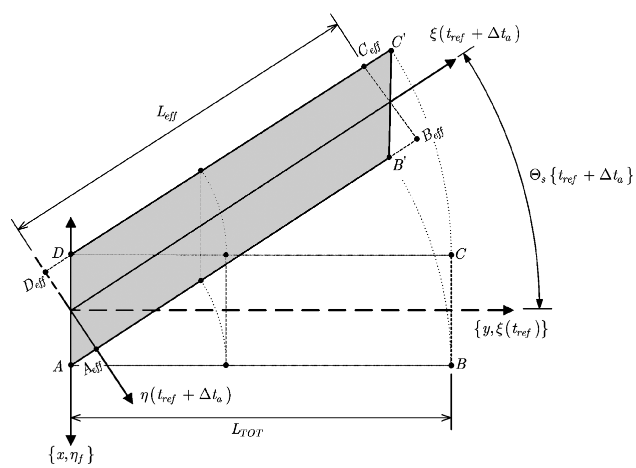
and

$$mr_{\mathrm{c.g.}}\frac{\mathrm{d}^2h}{\mathrm{d}t^2} + I_\alpha\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} + I_\alpha\omega_\theta^2\alpha = M \tag{4}$$

Thus, in Eqs. (3) and (4) the dependence of the coupled system dynamics is seen to be an *explicit* function of the uncoupled bending and torsional natural frequencies of the structure. Although Theodorsen's potential solution assumes an unswept wing of high aspect ratio to be represented by the given aerofoil at $\frac{3}{4}$ span, it is expected that a modified potential solution, taking into account the variability of the wing geometry, will exhibit similar functional dependence, hence, the impetus for our analytic efforts.



a) Three-dimensional view



b) Top view

Fig. 1 Illustrations of wing model.

III. Mathematical Model

A. Wing Description

An illustration of the proposed wing model is shown in Fig. 1. The $\{x, y\}$ plane is defined by the unit vectors $\{\hat{x}, \hat{y}\}$ originating from one-half the root chord length, extending parallel and perpendicular to the root chord, respectively, when the wing sweepback angle Θ_s is set to zero. The equality $\hat{z} = \hat{x} \times \hat{y}$ completes the vector triad and provides the basis for the airframe axis system. The relationship between the wing-fixed and airframe-fixed axis systems can be expressed in terms of an elementary rotation of magnitude Θ_s about $\{z, \chi\} : \tilde{T}^{WF} = R_z \tilde{T}^{AF}$, where $R_z \in \Re^{3 \times 3}$ is the familiar rotation matrix[13] and $\{\tilde{T}^{WF}, \tilde{T}^{AF}\} \in \Re^{3 \times 1}$ are matrices of unit vectors expressed in the wing-fixed and airframe-fixed coordinate systems, respectively. It should be evident that if $\Theta_S = 0$, $R_z = I$, and $\tilde{T}^{AF} = \tilde{T}^{WF}$, where I is the identity matrix.

The base of the wing root is assumed to remain confined to the $\{x, (z, \chi)\}$ plane during sweepback actuation, providing the geometric constraint that the trailing edge of the actuating wing should not pierce the wing-root/fuselage boundary. If the additional assumption is introduced that the bending and torsional stiffness of the root section (RS) are infinitely large at $\xi = 0 = \xi_{\text{RS}}$, the in-plane and out-of-plane deformations of the root section can be subsequently neglected.[14,15]

The piecewise defined actuation function $\boldsymbol{\Xi}^{WF}$ is introduced as a method to position the surface of the wing during a desired actuation cycle, expressed in the wing-fixed (WF) reference frame.

$$\boldsymbol{\Xi}^{WF}(\eta, \xi, \chi, t_A) = f_1(\eta, \xi, \chi, t_A)\hat{\boldsymbol{\eta}} + f_2(\eta, \xi, \chi, t_A)\hat{\boldsymbol{\xi}} + f_3(\eta, \xi, \chi, t_A)\hat{\boldsymbol{\chi}} \tag{5}$$

where

$$\{\eta | \eta_{\text{LE}} \le \eta \le \eta_{\text{TE}}, \xi | 0 < \xi \le \xi_{\text{TS}}, \chi | -\chi_l \le \chi \le \chi_u, t_A \ge 0\} \tag{6}$$

and the subscripts (LE, TE, TS, u, l) indicate the leading-edge and trailing-edge wing boundaries, the wing span, and the upper and lower wing boundaries, with respect to the wing mean line, respectively. The variable t_A refers to time in the sense of actuation of the wing geometry.

With the coordinate systems defined, an equivalent wing planform can now be considered. As a result of the wing-root/fuselage boundary condition already indicated, the sweptback planform will differ from the unswept (i.e., reference) planform. This can be seen in Fig. 1b by comparing $\square ABCD$ with $\square AB'C'D$. To correct for such geometric variation, an equivalent planform, given by $\square A_{\text{eff}} B_{\text{eff}} C_{\text{eff}} D_{\text{eff}}$, is employed. From Fig. 1b, the effective planform span along the ξ axis is seen to be given by $\xi_{\text{TS}} = L_{\text{eff}} = L\{\cos \Theta_s\}^{-1}$.

If $\boldsymbol{\Xi}^{WF}(\eta, \xi, \chi, t_A)$ and the material moduli $\{E(\eta, \xi, \chi, t_A), G(\eta, \xi, \chi, t_A), K(\eta, \xi, \chi, t_A)\}$ are known a priori, then the spanwise variation of the structural properties along the effective planform can be related to the reference planform via tensor transformation.[13]

To aid the reader, an example of this procedure can be taken from the simple sweepback actuation of the uniform wing of Fig. 1b. Assuming an isotropic material $E(\eta, \xi, \chi, t_A) \rightarrow E = \text{const}$, the effective bending stiffness can be calculated from the following development:

$$\boldsymbol{\Xi}^{WF}(\eta, \xi, \chi, t_A) \rightarrow \Theta_s = f(t_A)\hat{\boldsymbol{\chi}} \tag{7}$$

therefore,

$$R_z \rightarrow R_z(t_A) \, \forall \, t_A > 0 \quad \therefore \quad \chi(\eta, \xi, t_A) = \chi_{\text{RS}} \, \forall \, \{\xi | 0 < \xi \le \xi_{\text{TS}}, t_A \ge 0\} \tag{8}$$

and the bending moment of inertia becomes $I(\xi, t_A) \rightarrow I_{\text{RS}}$. Thus, using the sweepback kinematics given in Eq. (7), the effective bending stiffness at any instant t_A can be determined from

$$EI_{\text{eff}}(t_A) = E\left(R_z I_{\text{RS}} R_z^T\right) \forall \{\xi | 0 < \xi \le \xi_{\text{TS}}, t_A > 0\} \tag{9}$$

where $I_{\text{RS}} \in \Re^{3 \times 3}$ is the inertia tensor of second rank as expressed with respect to $\tilde{T}^{AF}$.

B. Morphable Wing Equations

With the wing model described, the development now focuses on the determination of the natural properties characteristic of the wing structure. The wing is assumed to be rigid perpendicular to the ξ coordinate direction. The additional constraint that the vibration of the wing is to be confined along a linear "average" elastic axis (EA) drawn through the average sectional shear centers along the span is also imposed. Consequently, for small sweepback angles, the governing partial differential equations can be elastically and inertially uncoupled. Although it is noted that this is not the case for most modern wing structures, this assumption is theorized to be reasonable for slender, thin planforms of relatively high aspect ratio. In terms of explicit values for the wing width-to-depth ratio, order-of-magnitude analysis suggests that this development is valid when the wing dimensions are selected such that $\{EI_b/GKL_{\text{eff}}^2, I_r/mL_{\text{eff}}^2\} \ll 1$, where EI_b and GK are the sectional bending and torsional stiffness, respectively.

The formulation of the governing equations of motion proceeds directly from the principle of superposition. Consider a linear homogeneous differential equation given by

$$\beta(\xi, D)y = 0 \tag{10}$$

where β represents a linear partial differential operator $|\mathcal{O}(\beta) = n$ in $\Re^{\vartheta}$, where $\vartheta = 1, 2, \ldots, \infty$. If a parameterized solution $\beta(\xi, D)y(\xi, \sigma)$ for a given value of σ in some interval $\Lambda \in \Re^{\vartheta \times \vartheta \times \cdots}$ is a solution to Eq. (10), the solution can be expressed as

$$\beta(\xi, D)y(\xi, \sigma) = 0 \, \forall \, \sigma \in \Lambda \tag{11}$$

Furthermore, if it is possible to obtain a real-valued function $\mathfrak{F}(\sigma)$ defined over the interval I, then the integral equation

$$y(\xi) = \int_{\Lambda} \mathfrak{F}(\sigma) y(\xi, \sigma)\, d\sigma \tag{12}$$

is also a solution to Eq. (10), provided that the integral appearing in Eq. (12) is convergent.[10] Taking into account the mathematical assumptions mentioned thus far, the use of Eq. (12) and considerable algebraic manipulation allows the partial differential equation pairs describing the bending (Φ^b) and torsional (Φ^θ) mode shapes of the wing model to be written as

$$\Phi^b\left(\xi_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) = (\omega^b)^2 \int_0^L g^b\left(\xi_{\mathrm{EA}}, \xi^*_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) m\left(\xi^*_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) \times \Phi^b\left(\xi^*_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) d\xi^*_{\mathrm{EA}} \tag{13}$$

$$\Phi^\theta\left(\xi_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) = (\omega^\theta)^2 \int_0^L g^\theta\left(\xi_{\mathrm{EA}}, \xi^*_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) I\left(\xi^*_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) \times \Phi^\theta\left(\xi^*_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) d\xi^*_{\mathrm{EA}} \tag{14}$$

where $\bar{L} = L(\boldsymbol{\Xi}^{\boldsymbol{WF}})\{\cos\Theta_s(\boldsymbol{\Xi}^{\boldsymbol{WF}})\}^{-1}$ and ξ_{EA} denotes that spanwise coordinates are expressed along the elastic axis. Given that the actuation dependant kernels of Eqs. (13) and (14) are one-dimensional Green's functions, note that the uncoupled morphable wing equations form a novel set of time-variant (in the actuation sense) Fredholm equations of the first kind.[10] Modifying the result of Ref. 9, derived from an application of Castigliano's theorem[15] to account for the variability of the wing geometry, material properties, and geometric properties, the required variable Green's functions can be expressed in the wing-fixed axis system as follows:

1) $\forall\, \xi^*_{\mathrm{EA}} \geq \xi_{\mathrm{EA}}$

$$g^b\left(\xi_{\mathrm{EA}}, \xi^*_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) = \int_0^{L^*} \frac{\left(\xi^*\mathfrak{A} - \mu_1\right)(\xi\mathfrak{A} - \mu_1)}{E\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}}\right) I_{\eta\eta}\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}}\right)}\, d\mu_1 + \int_0^{L^*} \frac{1}{G\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}}\right) K_{\xi_{\mathrm{EA}}}\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}}\right)}\, d\mu_1 \tag{15}$$

$$g^\theta\left(\xi_{\mathrm{EA}}, \xi^*_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) = \int_0^{L^*} \frac{\cos\Theta_s(\boldsymbol{\Xi}^{\boldsymbol{WF}})}{G\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}}\right) J_{\xi_{\mathrm{EA}}}\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}}\right)}\, d\mu_1 \tag{16}$$

2) $\forall\, \xi^*_{\mathrm{EA}} \leq \xi_{\mathrm{EA}}$

$$g^b\left(\xi_{\mathrm{EA}}, \xi^*_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) = \int_0^{L^{**}} \frac{\left(\xi^*\mathfrak{A} - \mu_1\right)(\xi\mathfrak{A} - \mu_1)}{E\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}}\right) I_{\eta\eta}\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}},\right)}\, d\mu_2 + \int_0^{L^{**}} \frac{1}{G\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}}\right) K_{\xi_{\mathrm{EA}}}\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}}\right)}\, d\mu_2 \tag{17}$$

$$g^\theta\left(\xi_{\mathrm{EA}}, \xi^*_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) = \int_0^{L^{**}} \frac{\cos\Theta_s(\boldsymbol{\Xi}^{\boldsymbol{WF}})}{G\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}}\right) J_{\xi_{\mathrm{EA}}}\left(\boldsymbol{\Xi}^{\boldsymbol{WF}}, \xi_{\mathrm{EA}}\right)}\, d\mu_2 \tag{18}$$

where $\mathfrak{A} = \{\cos\Theta_s(\boldsymbol{\Xi}^{\boldsymbol{WF}})\}^{-1}$, $L^{**} = \xi^*\mathfrak{A}$, and $L^* = \xi\{\cos\Theta_s(\boldsymbol{\Xi}^{\boldsymbol{WF}})\}^{-1}$. The variable $\mu_{1,2}$ appearing in Eqs. (15–18) is a variable of convolution arising from the expression of the observed deflection at a point ξ_{EA} as a result of a unit reference loading F (in the bending case) or unit reference moment M (in the torsional case) located at a point ξ^*_{EA} integrated $\forall\,\{\xi^*_{\mathrm{EA}}, \xi_{\mathrm{EA}}\} \in \Re_{\mathrm{EA}}$. Note that the boundary conditions necessary for the solution of Eqs. (13) and (14) are implicit within the kernels of the respective integral equations. Although omitted for the sake of brevity, the boundary conditions can be seen to be represented by the cantilevered beam[14] specialized in the sense that they must hold for all instances during the actuation cycle.

IV. Method of Solution

A. Mathematical Simplification

Equations (13) and (14) and their associated influence functions Eqs. (15–18) represent a formidable set of integral equations. Rather than considering a continuous wing structure, the problem can be further simplified by representing the wing as a collection n of discrete mass elements distributed along the assumed elastic axis. Clearly, the finer the mass distribution, the better the approximation to the continuous-wing model, that is, in the limit as $n \to \infty$ a continuous mass distribution is realized.

Considering the discrete mass model, Eqs. (13) and (14) can be recast in terms of matrix, rather than integral, notation.

$$\Phi_i\left(\xi_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) = \left(\omega_n^b\right)^2 \sum_{i=1}^{n} g^b_{i,k} m_i \Phi_k \to [\Phi] = \left(\omega_n^b\right)^2 [g^b][m][\Phi] \tag{19}$$

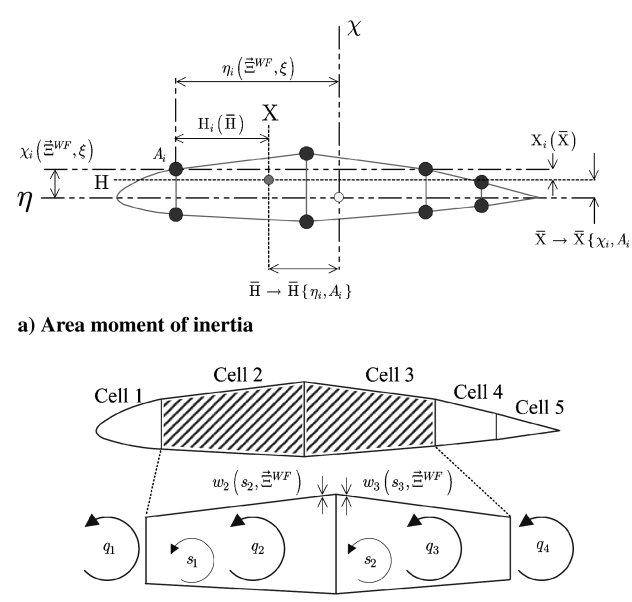
$$\theta_i\left(\xi_{\mathrm{EA}}, \boldsymbol{\Xi}^{\boldsymbol{WF}}\right) = \left(\omega_n^\theta\right)^2 \sum_{i=1}^{n} g^\theta_{i,k} I_i \theta_k \to [\theta] = \left(\omega_n^\theta\right)^2 [g^\theta][I][\theta] \tag{20}$$

where $[\Phi], [\theta] \in \Re^{n\times 1}$ are vectors of time-variant modal displacements; $[g^b], [g^\theta] \in \Re^{n\times n}$ are square matrices of time-variant influence coefficients as computed from Eqs. (15–18); $[m] \in \Re^{n\times n}$ is a positive, definite diagonal matrix of time-variant discrete mass values; and $[I] \in \Re^{n\times n}$ is a positive, definite diagonal matrix of time-variant mass moments of inertia. With all terms in Eqs. (19) and (20) known, with the exception of the modal displacements and natural frequencies, the unknown quantities can be solved using "brute-force" matrix iteration—particularly well suited for implementation on the digital computer.

B. Property Calculations

Thus far, the wing model has been defined, the required integral equations presented, and the numeric procedure for solving the discretized equations discussed. Now attention can be directed to the determination of time-variant sectional properties. Assuming a priori knowledge of the required actuation function, the geometric stiffness properties at each of the discrete mass elements can be calculated directly.

For example, at the jth spanwise mass the area moment of inertia is calculated according to the schematic shown in Fig. 2a. Assuming the shear panels of the wing to be ineffective in bending, with the stringer sections as the only contribution to the overall bending



a) Area moment of inertia

b) Torsional stiffness

Fig. 2 Schematics for variable property calculations.

rigidity, the centroidal location for the jth section is given by

$$\left\{\bar{H}\left(\xi_{\text{EA}}, \Xi^{WF}\right), \bar{X}\left(\xi_{\text{EA}}, \Xi^{WF}\right)\right\}$$
$$= \left\{\frac{\sum \eta_i\left(\Xi^{WF}, \xi_{\text{EA}}\right)A_i}{\sum A_i}, \frac{\sum \chi_i\left(\Xi^{WF}, \xi_{\text{EA}}\right)A_i}{\sum A_i}\right\} \tag{21}$$

With the jth centroidal location known as a function of the actuation parameter, the bending area moment of inertia is calculated from

$$I_b\left(\Xi^{WF}, \xi_{\text{EA}}\right) = \sum A_i X_i^2(\bar{X}) \tag{22}$$

For the torsional stiffness, the methods of Ref. 14 are modified to include the variation of the geometric and physical properties of the wing box during actuation. For this case, the stringers are assumed to contribute little to the overall torsional rigidity, thus limiting the analysis to the shear panels. Referring to Fig. 2b, the required expressions are identified as

$$1 = \sum 2A_i(w_i, s_i, \xi_{\text{EA}})q_i\left(\Xi^{WF}, A_i\right) \tag{23}$$

$$\theta_i\left(\Xi^{WF}, \xi_{\text{EA}}\right) = \frac{1}{2A_i(w_i, s_i, \xi_{\text{EA}})G\left(\Xi^{WF}, \xi_{\text{EA}}\right)} \oint_{\text{cell}\, i} \frac{q_i\left(\Xi^{WF}, A_i\right)}{w_i\left(s_i, \Xi^{WF}\right)} \, \mathrm{d}s_i \tag{24}$$

where the variables $\{w_i, s_i, q_i\}$ represent the thickness, boundary length and shear flow, respectively, of the ith cell. Introducing the compatibility relationship $\theta = \theta_1 = \theta_2 = \cdots = \theta_i$, Eqs. (23) and (24) can be solved simultaneously for the unknown twist angle θ and the torsion constant calculated for the jth location:

$$J_j\left(\Xi^{WF}, \xi_{\text{EA}}\right) = \frac{1}{G\left(\Xi^{WF}, \xi_{\text{EA}}\right)\theta\left(\Xi^{WF}, \xi_{\text{EA}}\right)} \tag{25}$$

The net result of this series of calculations is a collection of stiffness terms for every jth discrete location along the elastic axis of the form $z = \{a_1, a_2, \ldots, a_n\}^T$. Although the bending and torsional modal equations are simplified via discrete approximation, it is still desired to allow for a spanwise continuous variation of the bending (EI_b) and torsional (GJ) stiffness. Assuming a continuous basis function model $f(\xi)$ with unknown constant coefficients $\hat{x} = \{c_1, c_2, \ldots, c_k\}^T$ to be known, a least squares estimator[16] can be used to obtain the required property variation. Mathematically, using the stiffness vectrix z the basis coefficients can be found from

$$\hat{x} = (B^T B)^{-1} B^T z \tag{26}$$

where $B \in \Re^{n \times k} : n \geq k$ is an observation matrix constructed from the selected basis function. This method is advantageous in that experimental trends can be incorporated directly into the selection of the form of $f(\xi)$. Basis coefficients, resulting from evaluation of Eq. (26), can then be substituted into $f(\xi)$ to give the desired continuous material property variation, which can be used in Eqs. (15–18) to obtain the one-dimensional Green's functions necessary for the population of $\{[g^b], [g^\theta]\}$ and completing the described solution.

V. Conclusions

The main contribution of this Note is a novel technique for analyzing the modal properties of a morphable wing. A detailed description of the wing model was presented, and, based upon a given actuation function, the integral equations describing the modal properties of the model were presented along with the proposed method of solution. Additionally, the governing integral equations were shown to form a neoteric set of Fredholm equations, which are time dependant in the sense of the actuation variable. Although the method presented is specifically intended to be implemented in stability analyses of morphable aircraft with regard to self-excited oscillations, we would suggest this method for any investigation where the modal properties of such a structure are of concern.

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